Contribution to the Non-linear Properties of the SAW Transducers

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Abstract—**One of the important properties of the SAW transducer is the sensitivity to the external acting quantities. This sensitivity depends both on the strain and relative SAW velocity change due to the external, namely electric fields. The basic equations describing both contributions are presented in the paper. The resulting relative frequency change of the SAW transducer is the result of the strain due to the dimensions changes and the relative SAW velocity change due to the non-linear effects. The configuration of the oriented substrate (may be with domain wall), and the equations describing an influence of the electric field are described. Both the SC-cut quartz- and LiNbO3 substrates are discussed considered in this paper. The principle or new configuration may be used for SAW applications.**

I. Introduction

The electro-elastic effect in the BAW quartz resonators vibrating in the extensional mode, and influence of the strong external electric field on the resonant frequency was investigated by Hruska [1], [2]. The resulting effect was termed *polarization effect* and it was described by means of the change of elastic modules or coefficients caused by electric field. The influence of the electric field on the properties of piezoelectric medium was specified exactly by means of non-linear electro-elastic differential equations by Tiersten [3], [4], and the perturbation theory was presented by Baumhauer and Tiersten [5]. The equivalent treatment, i.e. using thermodynamically defined material constants for extensional mode resonators has been published by Kittinger and Tichy in [6]. In general, the perturbation method is used for calculation of the temperature dependence and acceleration effects both in bulk acoustic wave (BAW) and surface acoustic wave (SAW) devices.

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Lee and Yong [7] use this method for calculations of the temperature dependence of the resonant frequency of BAW resonators, however only the elastic non-linearities were considered in their treatment. Problem of the influence of external fields on the propagation of SAW and exactly description of the non-linenar properties of the SAW devices is a very complicated task. One of attempts to include all non-linearities into properties of SAW device was made by Kosek and Zelenka in your remarkable paper [8]. The influence of electric field on the SAW propagation on piezoelectric plate was described by non-linear terms in [8], [9], [10], [11], for example.
The surface acoustic wave (SAW) sensors of external fields as mechanical stress, strain, pressure, electric filed or temperature are based on the frequency change in SAW devices. The important parameter of SAW sensors is *sensor sensitivity* (per unit of the measured quantity), which is defined as the relative frequency change of the SAW oscillator or delay line. The relative frequency change $\Delta f/f_0$ depends both of the strain in the direction of the SAW propagation, and of the relative SAW velocity change $\Delta v/v$. Due to the piezoelectric effect and material non-linearities the external electric field changes both the dimensions of the SAW device and material parameters of a piezoelectric substrate. In our paper we suppose the SAW sensor of the strong external electric field, however the influence of the external stress is considered too. We suppose that the amplitude of the strain generated by the propagating SAW is small with respect to the strain created by an external electric field. Based on this condition, the SAW is considered as a small perturbation of the state of piezoelectric body, on which external field acts.
In general, the SAW propagation on the ferroelectric multidomain substrates is more complicated case in consequence of existence of some second order, non-linear effects due to the high amplitude of the SAW or subjected high electrostatic field to the SAW device. This electric field influences a velocity of the propagating and reflecting SAW [12].

II. Piezoelectric Substrate Subjected to the Electric Field

The three material states are considered in general non-linear theory [7]: natural, initial and final state. The initial state is defined by static and usually strong external fields, which acts on the SAW substrate. The final state is represented by strong electric filed, witch acts on the propagating SAW. Because the amplitude of the propagating SAW is small, the perturbation method can be used solving this state.

To describe the initial state, the influence of a strong electrostatic or slowly varying electric field acting to a SAW substrate can be defined by non-linear piezoelectric

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equations as components of the tensor of thermodynamic stress $t_{LM}^{\varepsilon,\sigma}$ (for constant thermodynamic intensity of electric filed ε and entropy $\boldsymbol{\sigma}$),

$$t_{LM}^{\varepsilon,\sigma} = \left[c_{LMPQ}^{\varepsilon,\sigma} + \frac{1}{2} c_{LMPQRS}^{\varepsilon,\sigma} \eta_{RS} - \frac{1}{2} e_{KLMPQ}^{\sigma} \varepsilon_{k} \right] \eta_{PQ} -$$

$$- \left[e_{KLM}^{\sigma} + \frac{1}{2} e_{KLMPQ}^{\sigma} \eta_{PQ} + \frac{1}{2} H_{KILM} \varepsilon_{I} \right] \varepsilon_{K} + t_{LM}^{(0)} \qquad (1)$$

and thermodynamic electric displacement

$$\mathsf{D}_{N}^{\eta,\sigma} = \left(e_{NAB}^{\sigma} + \frac{1}{2} e_{NABCD}^{\sigma} \eta_{CD} + H_{NMAB}^{\sigma} \varepsilon_{M} \right) \eta_{AB} +$$

$$\varepsilon_{NM}^{\sigma} \varepsilon_{M} + \frac{1}{2} \varepsilon_{NMA}^{\sigma} \varepsilon_{M} \varepsilon_{A} + \mathsf{D}_{N}^{(0)}, \qquad (2)$$

where external stress and electric displacement are included.

In (1) and (2) the symbol $\boldsymbol{\eta}_{PQ}$ denotes the components of final deformations, ε_{K} is tensor of thermodynamic electric field, symbol $c_{LMPQ}^{\varepsilon,\sigma}$ denotes the components of the tensor of linear elastic module (measured at constant thermodynamic intensity of electric field and entropy), $c_{LMPQRS}^{\varepsilon,\sigma}$ are the components of nonlinear elastic module, η_{RS} are the components of the tensor of thermodynamic deformation, e_{KLMPQ}^{σ} are components of the nonlinear piezoelectric-stress tensor (piezoelectric module), $\varepsilon_{K} = -\phi_{,K}$ is thermodynamic intensity of electric field and $\Phi_{,K}$ is derivation of electric potential according to the coordinate x_{K} .The symbols e_{KLM}^{σ} are components of the linear piezoelectric module, H_{KILM} components of the tensor of electrostriction coefficients, and ε_{NM} and ε_{NMP} are the components of the linear and non-linear permitivities.

The relations between the thermodynamic parameters $\mathsf{D}_{N}^{\eta,\sigma}$, ε_{N}, and D_{i}, ε_{i} in a space-coordinate x_{i} form are as follows:

$$D_N^{\eta,\sigma} = J \frac{\partial a_N}{\partial x_i} D_i, \qquad \text{and} \qquad \varepsilon_N = J \frac{\partial x_i}{\partial a_N} \varepsilon_i \tag{3}$$

where J is the Jacobian [13].

The following non-linear equation of motion (4) and electrical equation (5) can be used for determination of the elastic, or electrical quantities, respectively.

$$T_{LM} + T_{MN} u_{L,N} = \rho_0 \ddot{u}_L \tag{4}$$

$$D_{L,M}^{\eta,\sigma} = 0 \tag{5}$$

A shortened form of designation was used for derivatives.

If we compare (1) to the linear state equation for elastic stress T_{LM} , and suppose that amplitude of the oscillations of a cut is to small, the nonlinearity can be included into linear equations as small changes of linear material parameters [8]. Then we can express the change of the elastic module $c_{LMPQ}^{\varepsilon,\sigma}$ to the value c_{LMPQ}^{*}, and the piezoelectric module e_{KLM}^{σ} to the value e_{LMN}^{*}, due to the influence of electric field:

$$c_{LMPQ}^{*} = c_{LMPQ}^{\varepsilon,\sigma} - e_{KLMPQ}^{\sigma} \varepsilon_K \tag{6}$$

$$e_{KLM}^{*} = e_{KLM}^{\sigma} - \frac{1}{2} H_{KILM} \varepsilon_I \quad . \tag{7}$$

Because the elastic modules c_{LMPQ}^{*} and piezoelectric modules e_{LMN}^{*} are linear functions of electric field ε_K, the above mentioned modules will change by acting electric field and the amplitude of the change will be a linear function of the electric field. The modified material parameters may be substituted into linear SAW equations.

III. SAW VELOCITY CHANGE DUE TO THE ELECTRIC FIELD

As shown in (6) and (7), the electric field results in the change of elastic module in the area A, which is under electric field perpendicular to the propagation direction of SAW. At the same time, the length l_i of metal-coated part of the plate will be changed by the value Δl_i , as a result of piezoelectric effect. The proportional change of the length of metal-coated part of an area A can be derived from the deformation S_{AB} of a piezoelectric cut caused by electric field E_C

$$S_{AB} = d^*_{CAB}\, E_C \tag{8}$$

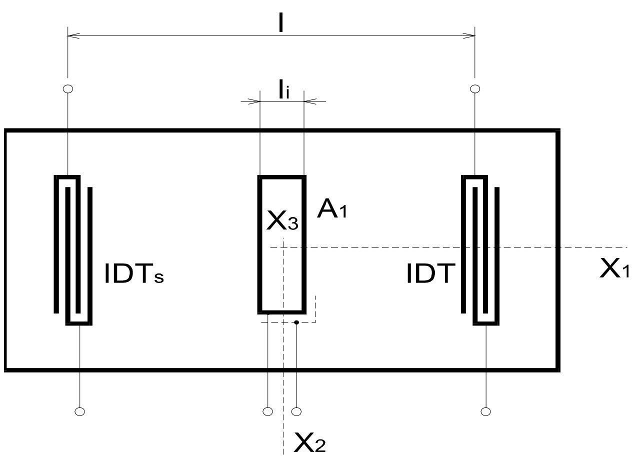

Fig. 1: Arrangement of control electrodes on piezoelectric substrate with interdigital transducers IDT_S and IDT

For arrangement of the plate according to the Fig.1 we obtain

$$\frac{\Delta l_i}{l_i} = d^*_{311} E_3 \ , \tag{9}$$

where d^*_{CAB} are components of tensor of piezoelectric coefficients defined in analogy to (7):

$$d^*_{CAB} = d^\sigma_{CAB} + \frac{1}{2} R_{CDAB}\, \varepsilon_D \quad . \tag{10}$$

In (10), the symbols R_{CDAB} are electrostriction coefficients. As a consequence of the change of relative length Δl_i, the SAW velocity changes from the value v to v' in the area A. The time τ necessary for SAW transmission between transmitting and receiving transducers is changed by $\Delta \tau_i$.
The velocity rate of surface acoustic waves propagation v and v' is a complicated function of linear and non-linear elastic modules $c^{\varepsilon,\sigma}_{LMPQ}$, c^*_{LMPQ} and piezoelectric modules e^σ_{KLM}, e^*_{KLM}. This ratio has to be determined by computer for the given orientation of a plate and the given directions of SAW propagation. In order to indicate further progress in solution, let us simply suppose, that the relations v and v', for the considered cut, are going to be simplified so that the velocity ratio can be expressed by a relation

$$\frac{v}{v'} = \left(\frac{c^{\varepsilon,\sigma}_{LMPQ}}{c^*_{LMPQ}} \right)^{\frac{1}{2}} \tag{11}$$

Using the relation (6), we get

$$\frac{v}{v'} = 1 + \frac{1}{2} \frac{e^\sigma_{KLMPQ}}{c^{\varepsilon,\sigma}_{LMPQ}} \varepsilon_K \tag{12}$$

where nonlinear piezoelectric coefficient e^S_{KLMPQ} can be expressed as

$$e^\sigma_{KLMPQ} = d^\sigma_{KFGH}\, c^{\varepsilon,\sigma}_{FGLM}\, c^{\varepsilon,\sigma}_{NHLM} + d^\sigma_{CAB}\, c^{\varepsilon,\sigma}_{BDLMPQ} \tag{13}$$

Using the procedure as in [14], we get to expression for relative time change of the delay line:

$$\frac{\Delta\tau_i}{\tau_i} = d_{CAB} - \frac{1}{2} R_{CDAB}\, \varepsilon_D - \frac{1}{2} \frac{e^{\sigma}_{KLMPQ}}{c^{\varepsilon,\sigma}_{LMPQ}} \varepsilon_K \tag{14}$$

The above relations prove that the surface acoustic wave time transmission can be changed by direct or slowly varying electric field.

IV. Results and Discussion

The quartz- and $LiNbO_3$ substrates were investigated in our laboratory. The quartz cuts, namely SC, have been studied by Kosek and Zelenka [8], the $LiNbO_3$ by Nosek [11], [14]. All cuts of quartz generate the maximum SAW relative velocity change less than 10 ppm. Because this change corresponds to the temperature change about 0.5K, only the SC-cuts (substrates with a zero temperature coefficient of SAW velocity) should bee considered in applications. On the SC-cut, the SAW group velocity is affected by the electric field parallel to the SAW beam axis. The maximum relative velocity change is therefore about 6 ppm. The $LiNbO_3$ (YZ-cut) substrates exhibit greater piezoelectric response than quartz. Also the sensitivity on the external electric field is greater than quartz. However, because the $LiNbO_3$ cuts have not a zero temperature coefficient, the influence of the temperature on relative SAW velocity may be greater than the influence of electric field. To realize the $LiNbO_3$ SAW delay line with greater sensitivity to the external electric field, the complicated structure based on two delay line structure was prepared. However, the condition of constant temperature is important. The last investigation of two domain substrate shows that phenomena of influencing external electric field, which is applied separately to both domains, considering propagation and reflections of the SAW, can be useful for signal processing.

V. Conclusion

The problem of SAW group velocity (Rayleigh waves) and its sensitivity to the external electric field can be linearized by modified material constants, using the perturbation method. The group velocity of the SAW propagating on the piezoelectric plate can by changed by external electric field, but the effect is less than the effect due to the temperature change. From this reasons, only the orientations of substrates with zero coefficient of SAW velocity are recommended for applications. The relative change of substrate dimension due to the piezoelectric effect must be considered. The two domain´s ferroelectric substrate with the domain wall considered here to be perpendicular to the SAW propagation can be useful for signal processing.

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